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Block-Wise Density Distribution of Primes Less Than A Trillion in Arithmetical Progressions $11n + k$

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Abstract: **Primes in arithmetical progressions $11n + k$ are considered for their comparative abundance in the ten power blocks of 1-10^n for $1 \le n \le 12$. In each of this block, the first and the last primes of respective forms are given. For inspecting scarcity of primes, minimum number of primes of these forms in these blocks, the first and the last blocks of minimality and their frequency within the limit of one trillion are determined. This analysis is also carried out for the maximal prime container blocks.**

Keywords: **Arithmetical progressions, block-wise distribution, prime, prime density.**

Mathematics Subject Classification 2010: **11A41, 11N05, 11N25.**

I. INTRODUCTION

In the fundamental branch number theory of mathematics, if any concept has been on the top priority for more than two millennia, it of prime numbers. Primes are known from quite long and their properties like infinitude have been explored with decent proof in the era as old as that of Euclid [1].

II. PRIMES NUMBERS AND ARITHMETICAL PROGRESSIONS

The combined study of prime numbers and arithmetical progressions is not new. On one hand, there are infinite primes and on the other, every arithmetical progression $an + b$ contains infinite number of numbers. The natural question is does it contain infinite number of primes? The answer is not always assertive. The condition under which the assertion holds was identified and proved by Dirichlet [2]. It says that progression $an + b$ can contain infinite number of primes if, and only if, a and b are co-primes, i.e., their greatest common factor is 1.

In this context all prime containing arithmetical progression $an + b$ with a = 2, 3, 4, 5, 6, 7, 8, 9, 10 have been extensively analyzed in earlier works [3]–[14]. There in addition to usual symbol $\pi(x)$ for number of primes less than or equal to x, a new symbol $\pi_{a,b}(x)$ has been introduced to mean the number of primes less than or equal to x in arithmetical progression $an + b$. Here we continue the use of that.

III. PRIMES DISTRIBUTIONS IN ARITHMETICAL PROGRESSIONS $11n + k$

In any arithmetical progression $an + b$, a and b are fixed integers, generally positive only. In fact, in standard arithmetical progressions, b is dependent on a, so long as its range is concerned. For any fixed a, the characteristic values of b range over 0, 1, . . . , $a - 1$. They together give a different arithmetical progressions which form a partition of the set of all integers I. That is these as sets of numbers in them are mutually exclusive and their union is I. Hence every integer is bound to be in one and only one of them, so is every prime. Of these, by Dirichlet's property, for those b's which have a common divisor greater than 1 with a, the progressions $an + b$ contain at most finite number of primes and others for which a and b have only 1 as the greatest common divisor, progression $an + b$ contain infinite number of primes.

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The case becomes special when a itself is a prime number. For now, except 0, every b with $1 \le b \le a - 1$ is co-prime with a and every progression $an + b$ $(1 \le b \le a - 1)$ contains infinite number of primes. Some cases like these have been seen earlier [3], [4], [6], [8]. Here we analyze first such two digit case of $a = 11$.

IV. PRIMES NUMBER RACE AMONGST THOSE IN PROGRESSIONS 11*n* + *k*

Generation of prime numbers has been challenge right from the beginning due to the formula they lack. There are various algorithms for this purpose. Time and effort saving prime generating algorithm is an outcome of an exhaustive comparison of many of them [15] – [21]. Implementing that over Java programming language [22], the analysis of this work was possible for all primes till one trillion.

Whenever the term prime number race comes in context of number of primes in arithmetical progressions, we cannot prevent temptation of recalling the mathematicians who introduced this terminology, viz., Granville and Martin [23], neither can one avoid such comparison of all valid progressions for understanding dominance of abundance of primes.

Since our $a = 11$ is itself prime here, there are as many as 10 progression candidates that contain infinitely many primes, viz., $11n + k$, for $k = 1, 2, 3, \ldots, 10$. Only one which contains finite number of primes, to be precise unique prime, is $11n + 0 = 11n$.

TABLE I: NUMBER OF PRIMES OF FORM 11*n* + *k* IN FIRST BLOCKS OF 10 POWERS

Sr. No.	Range 1-*x* (1 to *x*)	Number of Primes of Form				
		$11n + 1$ ($\pi_{11,1}(x)$)	$11n + 2$ ($\pi_{11,2}(x)$)	$11n + 3$ ($\pi_{11,3}(x)$)	$11n + 4$ ($\pi_{11,4}(x)$)	$11n + 5$ ($\pi_{11,5}(x)$)
1.	1-10	0	1	1	0	1
2.	1-100	3	3	2	2	2
3.	1-1,000	17	18	18	16	17
4.	1-10,000	125	120	122	118	121
5.	1-100,000	945	957	963	962	963
6.	1-1,000,000	7,858	7,843	7,814	7,839	7,853
7.	1-10,000,000	66,386	66,541	66,480	66,452	66,376
8.	1-100,000,000	576,103	576,332	575,872	575,818	576,332
9.	1-1,000,000,000	5,084,435	5,084,868	5,084,160	5,084,801	5,084,762
0.	1-10,000,000,000	45,504,543	45,506,100	45,503,956	45,505,446	45,505,736
1.	1-100,000,000,000	411,802,209	411,802,535	411,801,956	411,808,174	411,801,090
2.	1-1,000,000,000,000	3,760,794,629	3,760,792,712	3,760,791,888	3,760,781,586	3,760,794,820

Sr. No.	Range 1-*x* (1 to *x*)	Number of Primes of Form				
		$11n + 6$ ($\pi_{11,6}(x)$)	$11n + 7$ ($\pi_{11,7}(x)$)	$11n + 8$ ($\pi_{11,8}(x)$)	$11n + 9$ ($\pi_{11,9}(x)$)	$11n + 10$ ($\pi_{11,10}(x)$)
1.	1-10	0	1	0	0	0
2.	1-100	3	3	2	3	1
3.	1-1,000	18	17	15	15	16
4.	1-10,000	125	124	124	126	123
5.	1-100,000	966	955	958	953	969
6.	1-1,000,000	7,876	7,874	7,873	7,828	7,839
7.	1-10,000,000	66,448	66,490	66,507	66,425	66,473
8.	1-100,000,000	576,056	576,487	576,172	575,927	576,355
9.	1-1,000,000,000	5,085,277	5,084,752	5,085,005	5,084,213	5,085,260
0.	1-10,000,000,000	45,504,686	45,505,997	45,506,529	45,503,578	45,505,939
1.	1-100,000,000,000	411,809,535	411,807,927	411,806,854	411,806,740	411,807,792
2.	1-1,000,000,000,000	3,760,792,969	3,760,792,698	3,760,790,587	3,760,781,217	3,760,798,911

This has covered all primes till 1 trillion except 11 which is of form $11n + 0 = 11n$. Ignoring this, some forms like $11n + 10$ are ahead of average while others like $11n + 9$ mostly lag behind it for our discrete values, as shown in the figure.

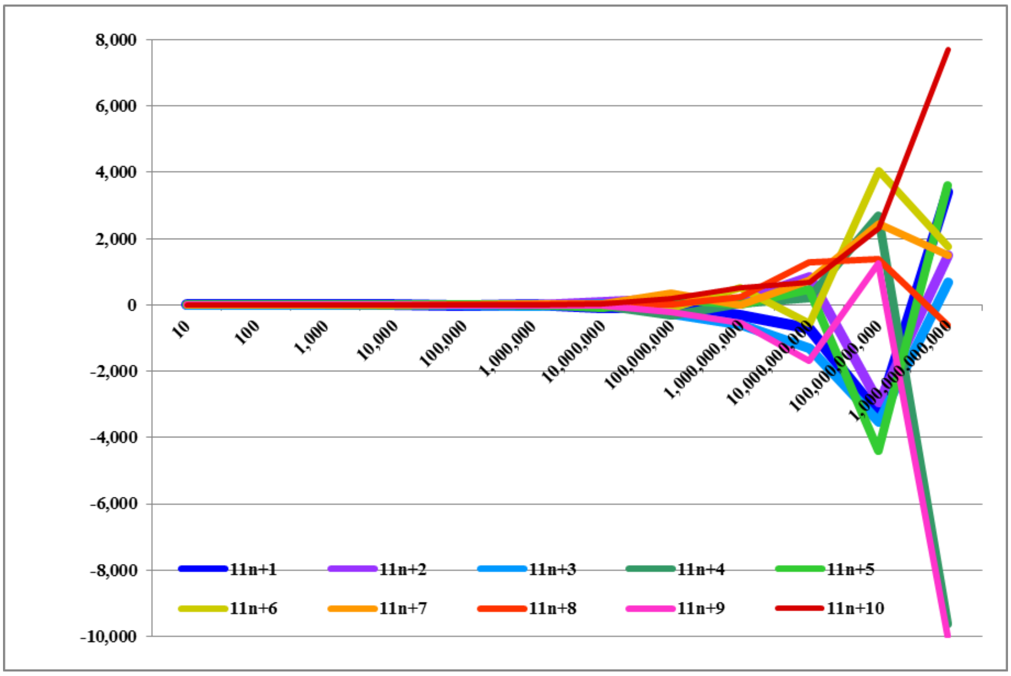

FIGURE I: DEVIATION OF $\pi_{11,k}(x)$ FROM AVERAGE.

V. BLOCK-WISE DISTRIBUTION OF PRIMES IN PROGRESSIONS $11n + k$

Our analysis has started block-wise and we continue it the same way. Instead of only first blocks of 10 powers, we consider all possible blocks of all possible 10 powers till 1 trillion. The blocks of various 10 powers that we get are :

1-10, 11-20, 21-30, 31-40, ⋯

1-100, 101-200, 201-300, 301-400, ⋯

1-1000, 1001-2000, 2001-3000, 3001-4000, ⋯

⋮

As our range is $1–10^{12}$, there come out 10^{12-n} number of blocks of 10^n size for each $1 \le n \le 12$. We refer to a block by a number one less that its starting; like for example, for 100 size, the block 0 means block of 1 – 100, block 100 means 101 – 200 etc.

5.1. The First and the Last Primes in the First Blocks of 10 Powers:

The quest of first primes in first blocks of all possible 10 powers is relatively easier as one gets them early.

TABLE II: FIRST PRIMES OF FORM $11n + k$ FIRST BLOCKS OF 10 POWERS

Sr. No	Range 1-x (1 to x)	First Prime in the First Block of form									
		$11n+1$	$11n+2$	$11n+3$	$11n+4$	$11n+5$	$11n+6$	$11n+7$	$11n+8$	$11n+9$	$11n+10$
1.	1-10	–	2	3	–	5	–	7	–	–	–
2.	1-100 till $1\text{-}10^{12}$	23	2	3	37	5	17	7	19	31	43

Cumbersome exercise of determination of last such primes of specific forms in first blocks has given the following.

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TABLE III: LAST PRIMES OF FORM 11*n* + *k* FIRST BLOCKS OF 10 POWERS

Sr. No	Range 1-*x* (1 to *x*)	Last Prime in the First Block of form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	–	2	3	–	5
2.	1-100	89	79	47	59	71
3.	1-1,000	991	937	971	983	929
4.	1-10,000	9,967	9,803	9,859	9,871	9,949
5.	1-100,000	99,991	99,871	99,971	99,961	99,929
6.	1-1,000,000	999,769	999,979	999,749	999,959	999,883
7.	1-10,000,000	9,999,991	9,999,937	9,999,971	9,999,653	9,999,973
8.	1-100,000,000	99,999,989	99,999,847	99,999,617	99,999,959	99,999,971
9.	1-1,000,000,000	999,999,353	999,999,937	999,999,883	999,999,587	999,999,929
10.	1-10,000,000,000	9,999,999,967	9,999,999,781	9,999,999,881	9,999,999,673	9,999,999,817
11.	1-100,000,000,000	99,999,999,947	99,999,999,871	99,999,999,289	99,999,999,851	99,999,999,907
12.	1-1,000,000,000,000	999,999,999,989	999,999,999,847	999,999,999,617	999,999,999,959	999,999,999,707

Sr. No	Range 1-*x* (1 to *x*)	Last Prime in the First Block of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	–	7	–	–	–
2.	1-100	83	73	41	97	43
3.	1-1,000	941	997	877	977	967
4.	1-10,000	9,851	9,973	9,941	9,931	9,833
5.	1-100,000	99,809	99,733	99,877	99,823	99,989
6.	1-1,000,000	999,983	999,907	999,853	999,953	999,613
7.	1-10,000,000	9,999,677	9,999,931	9,999,943	9,999,889	9,999,901
8.	1-100,000,000	99,999,587	99,999,643	99,999,941	99,999,931	99,999,547
9.	1-1,000,000,000	999,999,677	999,999,733	999,999,613	999,999,757	999,999,527
10.	1-10,000,000,000	9,999,999,851	9,999,999,929	9,999,999,787	9,999,999,557	9,999,999,943
11.	1-100,000,000,000	99,999,999,391	99,999,999,821	99,999,999,943	99,999,999,977	99,999,999,769
12.	1-1,000,000,000,000	999,999,999,961	999,999,999,863	999,999,999,611	999,999,999,359	999,999,999,899

It's time to compare both these parameters graphically to have a quick glimpse at trends.

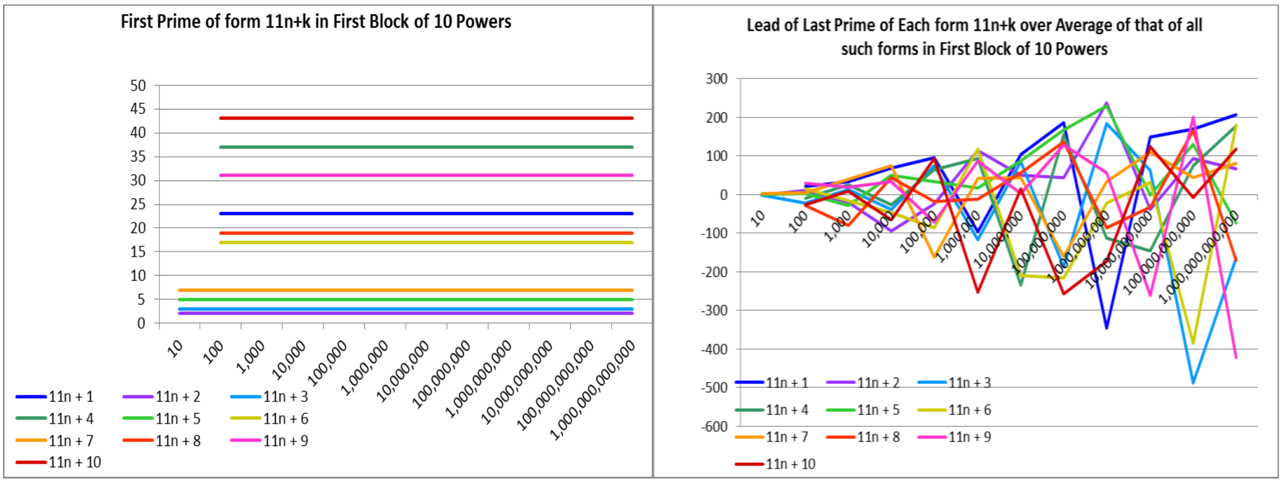

FIGURE II: FIRST & LAST PRIMES OF FORM 11*n* + *k* IN FIRST BLOCKS OF 10 POWERS.

And as mentioned earlier, the form 11*n* + 0 = 11*n* is kept aside in these; its first and last prime in all blocks of size 100 and higher are 11.

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5.2. Minimum Number of Primes in Blocks of 10 Powers:

The analysis of block-wise prime density demands the knowledge of minimum number of prime numbers within them for determining prime scarcity.

TABLE IV: MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN BLOCKS OF 10 POWERS

Sr. No	Range 1-*x* (1 to *x*)	Minimum Number of Primes in Blocks for form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	0	0	0	0	0
2.	1-100	0	0	0	0	0
3.	1-1,000	0	0	0	0	0
4.	1-10,000	11	12	11	12	11
5.	1-100,000	292	294	289	288	282
6.	1-1,000,000	3,429	3,446	3,445	3,429	3,423
7.	1-10,000,000	35,707	35,734	35,732	35,783	35,765
8.	1-100,000,000	361,186	361,042	360,971	361,056	361,173
9.	1-1,000,000,000	3,618,349	3,618,305	3,618,616	3,618,784	3,617,953
10.	1-10,000,000,000	36,202,825	36,201,252	36,193,050	36,201,446	36,195,480
11.	1-100,000,000,000	362,602,350	362,592,167	362,588,675	362,592,005	362,596,843
12.	1-1,000,000,000,000	3,760,794,629	3,760,792,712	3,760,791,888	3,760,781,586	3,760,794,820

Sr. No.	Range 1-*x* (1 to *x*)	Minimum Number of Primes in Blocks for form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	0	0	0	0	0
2.	1-100	0	0	0	0	0
3.	1-1,000	0	0	0	0	0
4.	1-10,000	12	12	12	10	12
5.	1-100,000	289	296	285	291	291
6.	1-1,000,000	3,434	3,427	3,441	3,420	3,422
7.	1-10,000,000	35,727	35,683	35,761	35,708	35,745
8.	1-100,000,000	361,052	361,071	361,190	361,126	361,117
9.	1-1,000,000,000	3,618,311	3,616,888	3,618,640	3,618,423	3,616,959
0.	1-10,000,000,000	36,194,731	36,193,977	36,201,252	36,198,536	36,194,872
1.	1-100,000,000,000	362,588,651	362,587,620	362,600,376	362,586,360	362,589,385
2.	1-1,000,000,000,000	3,760,792,969	3,760,792,698	3,760,790,587	3,760,781,217	3,760,798,911

The block-wise deviation of minimum number of primes from respective averages, except 11*n* + 0, comes ahead.

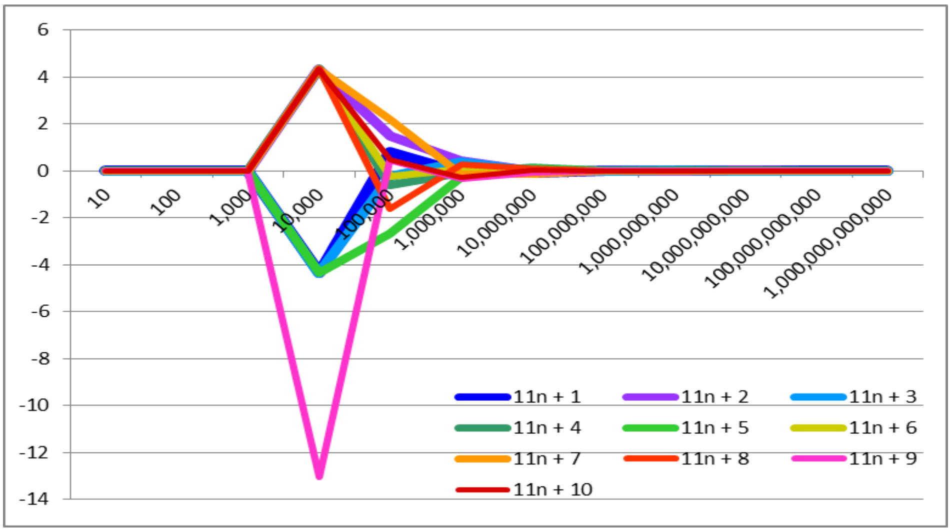

FIGURE III: % DEVIATION IN MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN BLOCKS OF 10 POWERS FROM AVERAGE

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With the convention adopted, now follow the first blocks with those many minimum number of primes in them.

TABLE V: FIRST BLOCKS OF 10 POWERS WITH MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No	Range 1-*x* (1 to *x*)	First Block with Minimum Number of Primes of form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	0	20	10	0	10
2.	1-100	200	300	1,000	1,300	1,400
3.	1-1,000	3,044,000	4,161,000	3,226,000	7,230,000	5,681,000
4.	1-10,000	867,275,260,000	434,307,190,000	711,562,010,000	427,226,920,000	567,923,030,000
5.	1-100,000	805,192,000,000	833,286,300,000	985,032,100,000	895,109,000,000	709,444,200,000
6.	1-1,000,000	945,771,000,000	941,148,000,000	875,402,000,000	956,827,000,000	790,270,000,000
7.	1-10,000,000	955,940,000,000	962,330,000,000	989,030,000,000	983,690,000,000	941,930,000,000
8.	1-100,000,000	992,200,000,000	997,000,000,000	995,200,000,000	989,700,000,000	972,200,000,000
9.	1-1,000,000,000	997,000,000,000	997,000,000,000	995,000,000,000	996,000,000,000	998,000,000,000
0.	1-10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
11.	1-100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

Sr. No	Range 1-*x* (1 to *x*)	First Block with Minimum Number of Primes of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	0	10	0	0	0
2.	1-100	1,800	1,500	400	500	3,400
3.	1-1,000	5,787,000	8,631,000	3,128,000	4,108,000	7,591,000
4.	1-10,000	631,546,300,000	412,284,570,000	547,550,600,000	865,552,140,000	533,845,650,000
5.	1-100,000	541,152,800,000	721,827,400,000	851,279,600,000	961,454,400,000	830,087,200,000
6.	1-1,000,000	987,380,000,000	908,327,000,000	991,702,000,000	779,726,000,000	928,439,000,000
7.	1-10,000,000	995,370,000,000	983,640,000,000	995,240,000,000	981,030,000,000	984,840,000,000
8.	1-100,000,000	997,000,000,000	994,300,000,000	996,500,000,000	999,500,000,000	980,200,000,000
9.	1-1,000,000,000	998,000,000,000	993,000,000,000	996,000,000,000	996,000,000,000	999,000,000,000
0.	1-10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
1.	1-100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

Similarly, the last blocks with those many minimum number of primes in them are also determined.

TABLE VI: LAST BLOCKS OF 10 POWERS WITH MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No	Range 1-*x* (1 to *x*)	Last Block with Minimum Number of Primes of form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	999,999,999,990	999,999,999,990	999,999,999,990	999,999,999,990	999,999,999,990
2.	1-100	999,999,999,700	999,999,999,900	999,999,999,900	999,999,999,800	999,999,999,900
3.	1-1,000	999,999,939,000	999,999,906,000	999,999,997,000	999,999,932,000	999,999,966,000
4.	1-10,000	984,749,900,000	933,769,560,000	711,562,010,000	795,643,980,000	963,976,010,000
5.	1-100,000	805,192,000,000	986,615,800,000	985,032,100,000	973,060,600,000	709,444,200,000
6.	1-1,000,000	945,771,000,000	941,148,000,000	875,402,000,000	956,827,000,000	790,270,000,000
7.	1-10,000,000	955,940,000,000	962,330,000,000	989,030,000,000	983,690,000,000	941,930,000,000
8.	1-100,000,000	992,200,000,000	997,000,000,000	995,200,000,000	989,700,000,000	972,200,000,000
9.	1-1,000,000,000	997,000,000,000	997,000,000,000	995,000,000,000	996,000,000,000	998,000,000,000
0.	1-10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
1.	1-100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

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Sr. No	Range 1-x (1 to x)	Last Block with Minimum Number of Primes of form				
		11n + 6	11n + 7	11n + 8	11n + 9	11n + 10
1.	1-10	999,999,999,990	999,999,999,990	999,999,999,990	999,999,999,990	999,999,999,990
2.	1-100	999,999,999,800	999,999,999,900	999,999,999,900	999,999,999,900	999,999,999,900
3.	1-1,000	999,999,896,000	999,999,993,000	999,999,992,000	999,999,942,000	999,999,984,000
4.	1-10,000	765,033,460,000	870,788,820,000	688,140,350,000	865,552,140,000	875,889,070,000
5.	1-100,000	541,152,800,000	989,254,500,000	851,279,600,000	961,454,400,000	830,087,200,000
6.	1-1,000,000	987,380,000,000	908,327,000,000	991,702,000,000	779,726,000,000	928,439,000,000
7.	1-10,000,000	995,370,000,000	983,640,000,000	995,240,000,000	981,030,000,000	984,840,000,000
8.	1-100,000,000	997,000,000,000	994,300,000,000	996,500,000,000	999,500,000,000	980,200,000,000
9.	1-1,000,000,000	998,000,000,000	993,000,000,000	996,000,000,000	996,000,000,000	999,000,000,000
0.	1-10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
1.	1-100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

We compare both types graphically.

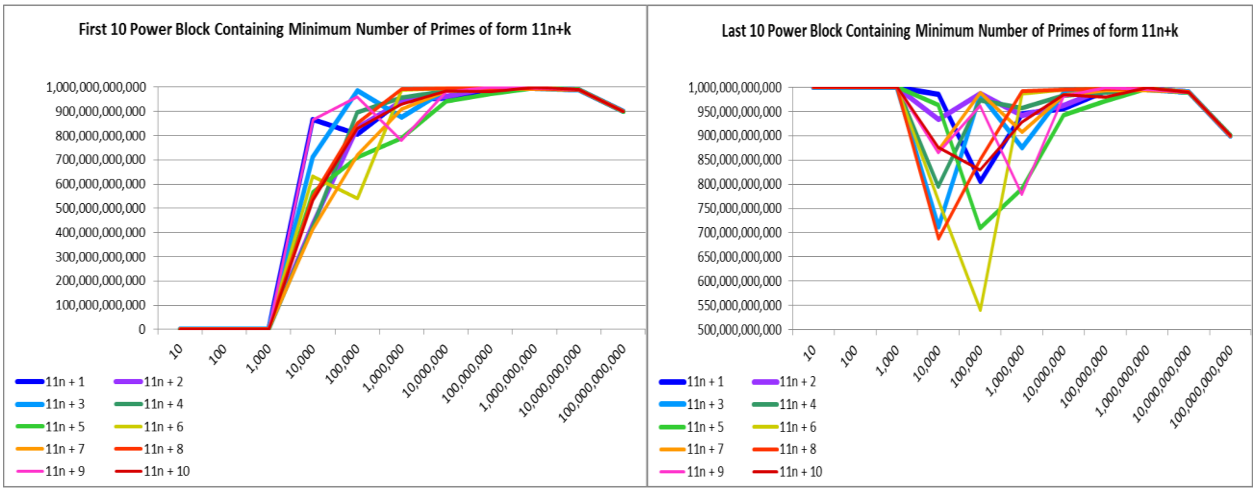

FIGURE IV: FIRST & LAST BLOCKS OF 10 POWERS WITH MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k*.

The unmentioned values for forms 11n + 0 are parallel to corresponding values for arithmetical progression 8n + 2 given in [9]. Next search was of the number of times such blocks with minimum number of primes of all these forms occur.

TABLE VII: FREQUENCY OF 10 POWER BLOCKS WITH MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No	Range 1-x (1 to x)	No. of Blocks with Minimum No. of Primes of form				
		11n + 1	11n + 2	11n + 3	11n + 4	11n + 5
1.	1-10	96,239,205,371	96,239,207,288	96,239,208,112	96,239,218,414	96,239,205,180
2.	1-100	6,670,644,133	6,670,670,372	6,670,672,363	6,670,677,164	6,670,645,505
3.	1-1,000	13,898,378	13,905,293	13,900,677	13,900,419	13,899,232
4.	1-10,000	4	3	1	5	2
5.	1-100,000	1	4	1	2	1
6.	1-1,000,000 till 10^{11}	1	1	1	1	1

Sr. No	Range 1-*x* (1 to *x*)	No. of Blocks with Minimum No. of Primes of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	96,239,207,031	96,239,207,302	96,239,209,413	96,239,218,783	96,239,201,089
2.	1-100	6,670,672,703	6,670,645,889	6,670,675,043	6,670,667,440	6,670,676,692
3.	1-1,000	13,899,087	13,899,259	13,900,091	13,891,822	13,898,180
4.	1-10,000	2	5	3	1	2
5.	1-100,000	1	5	1	1	1
6.	1-1,000,000 till 10^{11}	1	1	1	1	1

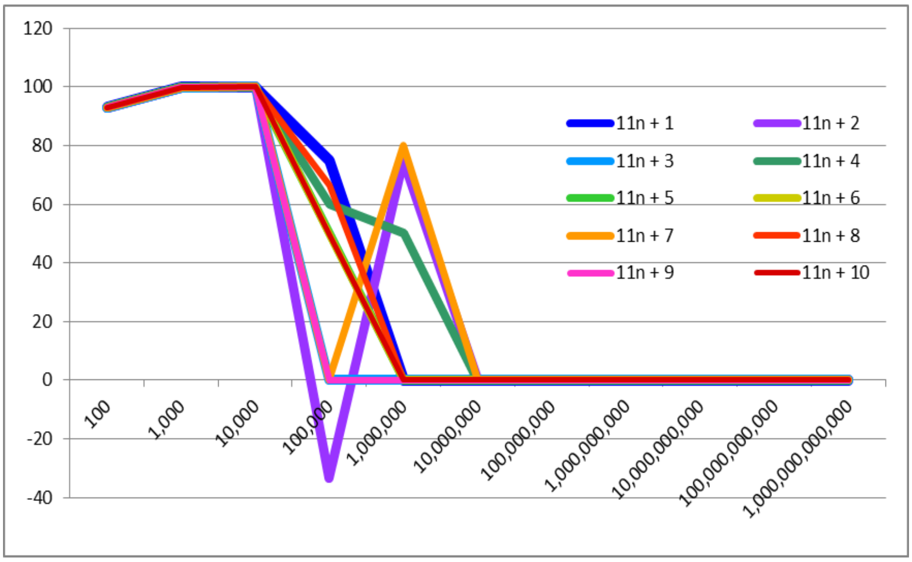

FIGURE V: % DECREASE IN OCCURENCES OF MINIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN BLOCKS OF 10 POWERS.

5.3. Maximum Number of Primes in Blocks of 10 Powers:

The analysis of block-wise prime density also demands the knowledge of maximum number of prime numbers within them for determining prime abundance.

TABLE VIII: MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN BLOCKS OF 10 POWERS

Sr. No	Range 1-*x* (1 to *x*)	Maximum Number of Primes in Blocks for form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	1	1	1	1	1
2.	1-100	4	4	4	4	4
3.	1-1,000	17	18	18	17	17
4.	1-10,000	125	120	122	118	121
5.	1-100,000	945	957	963	962	963
6.	1-1,000,000	7,858	7,843	7,814	7,839	7,853
7.	1-10,000,000	66,386	66,541	66,480	66,452	66,376
8.	1-100,000,000	576,103	576,332	575,872	575,818	576,332
9.	1-1,000,000,000	5,084,435	5,084,868	5,084,160	5,084,801	5,084,762
10.	1-10,000,000,000	45,504,543	45,506,100	45,503,956	45,505,446	45,505,736
11.	1-100,000,000,000	411,802,209	411,802,535	411,801,956	411,808,174	411,801,090
12.	1-1,000,000,000,000	3,760,794,629	3,760,792,712	3,760,791,888	3,760,781,586	3,760,794,820

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Sr. No.	Range 1-*x* (1 to *x*)	Maximum Number of Primes in Blocks for form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	1	1	1	1	1
2.	1-100	4	4	4	4	4
3.	1-1,000	18	17	17	16	16
4.	1-10,000	125	124	124	126	123
5.	1-100,000	966	955	958	953	969
6.	1-1,000,000	7,876	7,874	7,873	7,828	7,839
7.	1-10,000,000	66,448	66,490	66,507	66,425	66,473
8.	1-100,000,000	576,056	576,487	576,172	575,927	576,355
9.	1-1,000,000,000	5,085,277	5,084,752	5,085,005	5,084,213	5,085,260
10.	1-10,000,000,000	45,504,686	45,505,997	45,506,529	45,503,578	45,505,939
11.	1-100,000,000,000	411,809,535	411,807,927	411,806,854	411,806,740	411,807,792
12.	1-1,000,000,000,000	3,760,792,969	3,760,792,698	3,760,790,587	3,760,781,217	3,760,798,911

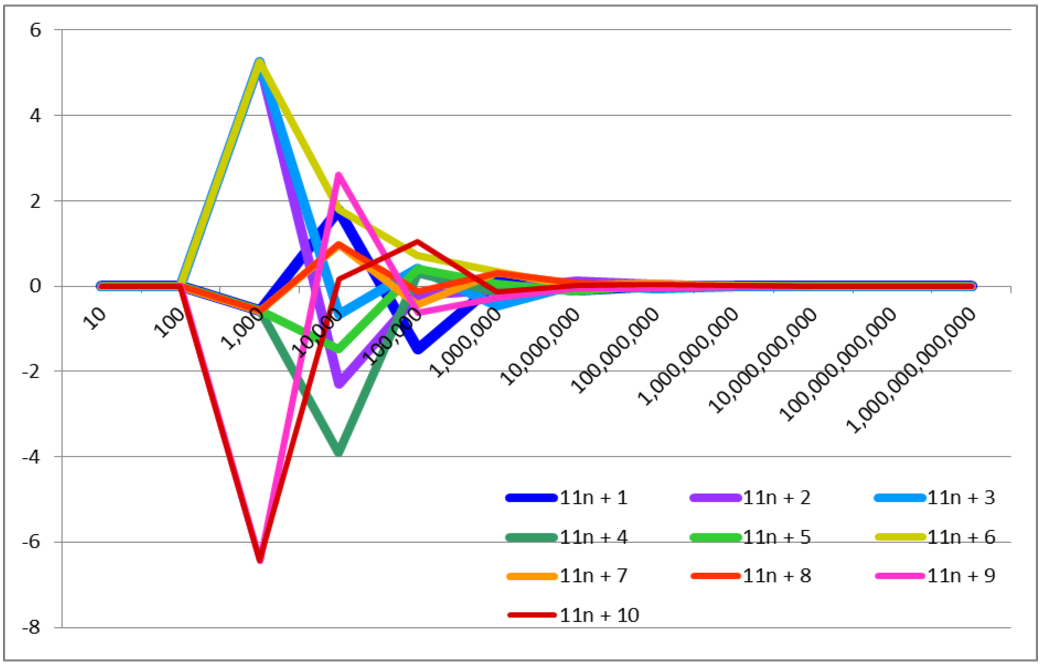

FIGURE VI: % DEVIATION IN MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN BLOCKS OF 10 POWERS FROM AVERAGE.

Like minimum analysis, now the first blocks containing maximum number of such primes are determined.

TABLE IX: FIRST BLOCKS OF 10 POWERS WITH MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No.	Range 1-*x* (1 to *x*)	First Block with Maximum Number of Primes of form									
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5	11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	20	0	0	30	0	10	0	10	30	40
2.	1-100	2,300	21,000	67,400	319,400	17,000	52,200	35,900	1,800	3,000	49,400
3.	1-1,000	0	0	0	4,000	0	0	0	1,000	1,336,000	0
4.	1-10,000 till $1\text{-}10^{12}$	0	0	0	0	0	0	0	0	0	0

Likewise, the last blocks with maximum number of primes in them are found to be as follows.

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TABLE X: LAST BLOCKS OF 10 POWERS WITH MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No	Range 1-*x* (1 to *x*)	Last Block with Maximum Number of Primes of form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	999,999,999,980	999,999,999,840	999,999,999,610	999,999,999,950	999,999,999,700
2.	1-100	999,998,511,600	999,997,928,700	999,999,220,200	999,999,617,400	999,995,594,900
3.	1-1,000	0	0	0	4,000	0
4.	1-10,000 till 10^{12}	0	0	0	0	0

Sr. No	Range 1-*x* (1 to *x*)	Last Block with Maximum Number of Primes of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	999,999,999,960	999,999,999,860	999,999,999,610	999,999,999,350	999,999,999,890
2.	1-100	999,996,081,200	999,991,730,700	999,998,312,100	999,991,431,800	999,993,021,300
3.	1-1,000	0	0	1,000	1,336,000	642,324,005,000
4.	1-10,000 till 10^{12}	0	0	0	0	0

Due to decreasing density of primes, it is no surprise that soon the block 0, corresponding to the starting block is the only block containing maximum primes. So we restrict our graph limits of blocks to only first few to focus on initial comparisons.

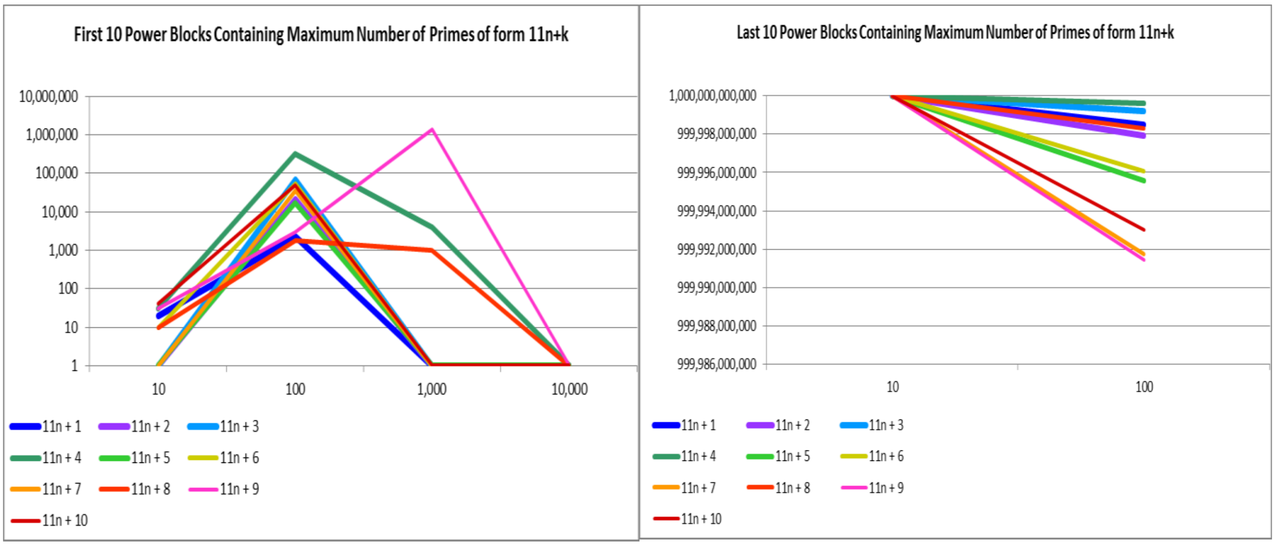

FIGURE VII: FIRST & LAST BLOCKS OF 10 POWERS WITH MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k*

The same decreasing frequency of primes forces frequency of maximum prime containing blocks to be 1 soon.

TABLE XI: FREQUENCY OF 10 POWER BLOCKS WITH MAXIMUM NUMBER OF PRIMES OF FORM 11*n* + *k* IN THEM

Sr. No	Range 1-*x* (1 to *x*)	No. of Blocks with Maximum No. of Primes of form				
		11*n* + 1	11*n* + 2	11*n* + 3	11*n* + 4	11*n* + 5
1.	1-10	3,760,794,629	3,760,792,712	3,760,791,888	3,760,781,586	3,760,794,820
2.	1-100	239,253	240,009	239,405	239,571	239,900
3.	1-1,000 till 10^{12}	1	1	1	1	1

Sr. No.	Range 1-*x* (1 to *x*)	No. of Blocks with Maximum No. of Primes of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
1.	1-10	3,760,792,969	3,760,792,698	3,760,790,587	3,760,781,217	3,760,798,911
2.	1-100	240,477	239,772	239,424	239,907	240,610

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Sr. No.	Range 1-*x* (1 to *x*)	No. of Blocks with Maximum No. of Primes of form				
		11*n* + 6	11*n* + 7	11*n* + 8	11*n* + 9	11*n* + 10
3.	1-1,000	1	1	1	1	3
4.	1-10,000 till 10^{12}	1	1	1	1	1

The unconsidered arithmetical progression 11*n* + 0 contains maximum 1 prime only once for which the first and the last block of occurrence is the very first block 0 starting with block size 100 onwards, the frequency of which is 1.

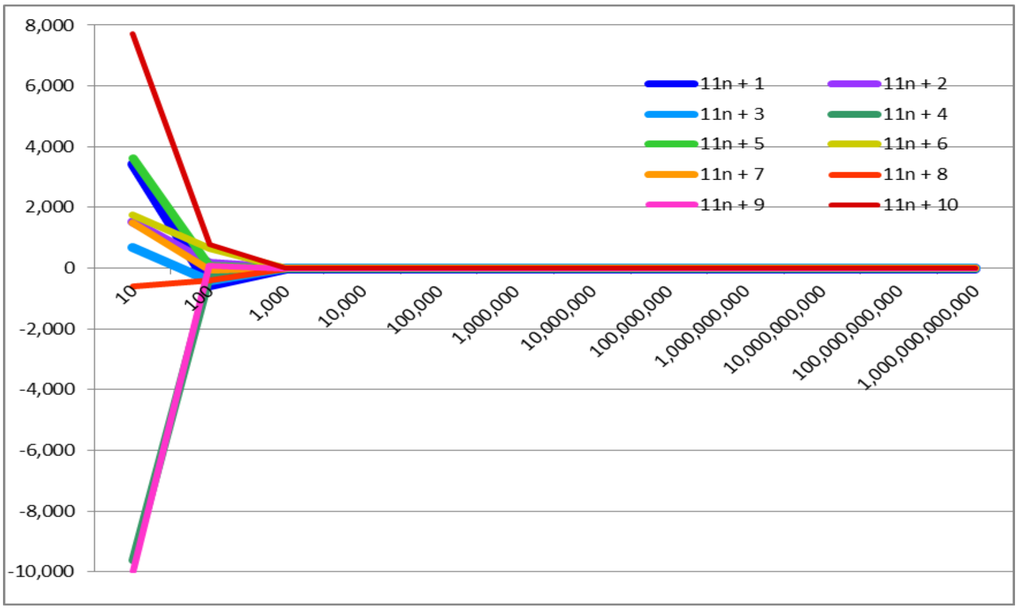

FIGURE VIII: DEVIATION IN FREQUENCY OF MAXIMUM NUMBER OF PRIMES IN BLOCKS FROM AVERAGE

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